REMARKS

Claims 2-36 are pending in this application.

Claims 4-7, 13-16, 20-22, 26-28 and 32-34 are allowed.

Claims 2-3, 8-12, 17-19, 23-25, 29-31 and 35-36 are rejected.

The final office action dated July 15, 2004 indicates that claims 17-19 and 36 are rejected under 35 USC §112, second paragraph, as being indefinite because claim 17 lacks antecedent basis for "the mapping function." This rejection has been overcome by the amendment above to claim 17. Claim 19 has been amended to depend properly from amended claim 17.

The final office action indicates that claims 10, 17 and 36 are rejected under 35 USC §102(b) as being anticipated by an article by Zimmerman et al. ("An Evaluation of the Effectiveness of Adaptive Histogram Equalization for Contrast Enhancement"); claim 29 is rejected under 35 USC §102(b) as being anticipated by Lee U.S. Patent No. 5,361,308; claims 2, 3 and 35 are rejected under 35 USC §103(a) as being unpatentable over Zimmerman et al. in view of Lee; and claims 23-25 are rejected under 35 USC §103(a) as being unpatentable over Burke U.S. Patent No. 5,042,077 in view of Zimmerman et al. Dependent claims 8-9, 11-12, 18-19 and 30-31 are also rejected. Only the rejections of base claims 2, 10, 17, 23, and 29 will be discussed below.

'103 rejection of claim 29

Claim 29 recites an article for a processor. The article comprises memory; and an image sharpening program stored in the memory. The program, when executed, causes the processor to process pixels of interest. Each pixel of interest is processed by clipping its intensity value if its intensity value lies outside of a variable contrast range, and mapping its intensity value if its intensity value lies within the variable contrast range.

Lee relates to three-dimensional measurement of object profiles using machine vision (col. 1, lines 5-8). Stripes of light are projected onto a tool surface, and a resulting stripe pattern, which varies sinusoidally, is imaged by a CCD camera (col. 3, lines 1-7 and col. 4, lines 35-38). The raw image is enhanced at block 32 of Figure 3 (col. 5, lines 1-2). The image enhancement is performed to improve the ability of finding stripes in the image.

The result of the image enhancement is to blur the image (col. 5, line 33). However, the stripes have greater intensity (col. 5, line 34). Thus, Lee does not teach or suggest an image sharpening program.

The office action argues that "image sharpening" in claim 29 carries no patentable weight because it is recited in the preamble and not the body of the claim. This argument might apply to claim 10, but not to claim 29. The body of claim 29 recites "an image sharpening program."

Moreover, the enhancement at block 32 does not use a contrast range that is variable. Col. 5, lines 22-29, of Lee states the following.

> Histogram scaling is applied to expand the range of intensity levels to fill the entire available range. For the transformation function, a simple linear ramp is used with two clip points. A program 25 calculates the fractions for low clipping from the background and for high clipping from the object area. It roughly relates to clipping the entire background to 0.0 and taking the brightest 5% of the object to clip to 1.0.

Thus, Lee determines only two clip points and a single linear ramp between those two clip points. Lee clips the **<u>entire</u>** background to 0.0. Lee clips to the brightest 5% of the object. Lee does not teach or suggest that the clip points or ramp are variable in any way.

The office action argues that the clip points vary according to the intensity levels of the background and object area. The clip points might vary from image

to image, but they don't vary within the same image. Claim 29 recites a contrast range that is variable within the image being processed.

For these reasons, Lee does not teach or suggest the article of claim 29. Therefore, claim 29 and its dependent claims 30-31 should be allowed over Lee.

'103 rejection of claim 10

Claim 10 recites a method of sharpening a digital image that includes a plurality of pixels of interest. For each pixel of interest the method includes comprising determining a dynamic range of a pixel neighborhood, where the dynamic range of a pixel neighborhood is based on a difference of minimum and maximum pixel values in the pixel neighborhood; and performing contrast stretching according to the corresponding dynamic range.

Zimmerman et al. disclose a histogram equalization method. Histogram equalization, in general, is a well known technique for contrast enhancement. Each gray level G is counted to determine the number of times it appears in the image. The counts are normalized by the total number of pixels to obtain an approximated probability of encountering each gray-level in a selected region: P(G)=N(G)/total_N. Once the histogram P(G) is computed, a histogram equalization mapping M is defined as the cumulative sum of the histogram, i.e. M(G) = sum(P(G'),for all G'≤G). This guaranties that M(0)=0, M(1)=1, and that M(G) is non-decreasing. For further information about histogram equalization, see the following two attached references: Gonzalez et al., "Digital Image processing." ISBN 0-201-18075-8, pp. 91-94 (2002); and "Histogram-based operations" at http://www.ph.tn.tudelft.nl/Courses/FIP/noframes/fip-istogram.html.

In general, histogram equalization is not suited for image sharpening. Histogram equalization mapping tends to spread the local output histogram, thereby blurring edges instead of sharpening them.

Zimmerman et al. modify this well known technique by choosing a small number of contextual regions within an image and calculating a mapping for a given pixel as a bilinear interpolation of the mappings derived from nearby contextual regions. The modification is supposed to make the histogram equalization faster to perform.

In Zimmerman et al.'s method, a dynamic range is not determined for each pixel. Zimmerman et al. do not even compute a histogram equalization mapping for each pixel, since the computational burden would be too great. Instead Zimmerman et al. estimate for each pixel a mapping which ***approximates*** the histogram equalization mapping for that pixel's neighborhood. The estimation is based on nearby histogram equalizations for other pixels that were actually computed.

The method of claim 10 does not involve computing a histogram equalization mapping for each pixel, nor does it involve computing a histogram equalization for a number of regions. The method of claim 10 is much simpler, as it involves computing dynamic ranges (e.g., differences between minimum and maximum pixel values). Thus the method of claim 10 is far less computationally intensive than computing histogram equalizations.

The office action relies on a single paragraph in Zimmerman et al. (page 305, left column, final paragraph) and argues the following.

> With regard to claim 10, Zimmerman discloses determining a dynamic range of a pixel neighborhood, where the dynamic range of a pixel neighborhood is based on a difference of minimum and maximum pixel values in the pixel neighborhood; and then performing contrast stretching on a pixel-by-pixel basis according to the corresponding dynamic range (Zimmerman pg. 305 left column, final paragraph). The "distribution of pixel intensities" as disclosed in Zimmerman is analogous to a dynamic range based on a difference of minimum and maximum pixel values as recited in the claim. The difference between the minimum and maximum pixel

> values of a region is well known in the art as the distribution of pixel intensities for the region. Applying the "histogram equalization mapping" as disclosed in Zimmerman is analogous to performing contrast stretching as recited in the claim.

This argument is confusing. The office action asserts that the distribution of pixel intensities is <u>analogous</u> to a dynamic range, and that the histogram equalization is merely <u>analogous</u> to contrast stretching. Is the examiner arguing that claim 10 is anticipated by Zimmerman et al, because Zimmerman et al. disclose analogous teachings, or is the examiner arguing that claim 10 is anticipated by Zimmerman et al, because Zimmerman et al. disclose identical teachings?

According to MPEP 2131, "'A claim is anticipated only if each and every element as set forth in the claim is found, either expressly or inherently described, in a single prior art reference'.... 'The identical invention must be shown in as complete detail as is contained in the ... claim.'"

Analogous art is a separate issue. The examiner must determine whether prior art is "analogous" for the purpose of analyzing the obviousness of the subject matter at issue. See MPEP 2141.01(a). To base a '102 or '103 rejection in view of Zimmerman et al., Zimmerman et al. must be analogous art. Whether Zimmerman et al is analogous art is not at issue.

The issue is whether Zimmerman et al. discloses every element as set forth in the claim 10. If the term "analogous" is taken at face value, the office action does not even contend that claim 10 is anticipated by Zimmerman et al.

If the examiner confuses "analogous" with "identical", then the rejection still falls short. The rejection of claim 10 would be based on the following assertions:

1. The distribution of pixel intensities is identical to a dynamic range based on a difference of minimum and maximum pixel values.

2. The difference between the minimum and maximum pixel values of a region is well known in the art as the distribution of pixel intensities for the region.

3. Histogram equalization and contrast stretching are identical.

The applicant, who is skilled in the art of image processing, is not familiar with these assertions. First, assertion (1) is the mirror of assertion (2), and both are inaccurate. The distribution at issue is a probability function of a pixel value occurring, not a difference between two values.

Second, assertion (3) is little more than an unsubstantiated conclusion. The conclusion can be substantiated by a side-by-side comparison of the steps performed during histogram equalization and the steps performed during contrast stretching. Thus far, no analysis has been provided.

Third, no documents are cited to support these assertions. Therefore, these assertions are presumed to be within the personal knowledge of the examiner. Pursuant to MPEP §707 and 37 CFR §1.104(d)(2), the examiner is respectfully requested to cite a document or affidavit supporting his personal knowledge.

Thus far, the documents made of record do not support a '102 or '103 rejection. Therefore, claim 10 and its dependent claims 11-12 should be allowed over the documents made of record.

Finally, the office action erroneously dismisses the weight of the preamble. The office action states a general rule for disregarding the weight of a preamble. However, MPEP 2111.02 states

> The determination of whether a preamble limits a claim is made on a case-by-case basis in light of the facts in each case; there is no litmus test defining when a preamble limits the scope of a claim.... If the claim preamble, when read in the context of the entire claim, recites limitations of the claim, or, if the claim preamble is 'necessary to give life, meaning, and vitality' to the claim, then the claim preamble should be construed as if in the balance of the claim.

The preamble of claim 10 is quite clear that the method applies to image sharpening. In claim 10, image sharpening is not one of several possible uses.

There is no teaching or suggestion that Zimmerman et al.'s method can be used for image sharpening. Zimmerman et al.'s method of local histogram equalization is suited for correcting uneven illumination in images. It is not suited for image sharpening because it tends to spread the local output histogram, thereby blurring edges instead of sharpening them. For this additional reason, claim 10 and its dependent claims 11-12 should be allowed over Zimmerman et al.

'103 rejection of claim 17

Claim 17 recites apparatus for processing pixels of interest in a digital image. The apparatus comprises a processor for determining dynamic ranges of pixel neighborhoods for the pixels of interest, and applying a contrast stretching function to each pixel of interest within the dynamic range of the corresponding pixel neighborhood. The contrast stretching function has a shape that depends on the dynamic range.

As indicated above, Zimmerman et al. do not teach or suggest determining a dynamic range. For this reason alone, claim 17 and its dependent claims 18-19 and 36 should be allowed over Zimmerman et al.

The office action asserts that "contrast enhancement mapping disclosed in Zimmerman is analogous to the contrast stretching function recited in the claim." First, the issue of "analogous" is irrelevant, for the real Issue is whether Zimmerman et al. disclose the teachings of claim 10. Second, there is no support I the prior art for this assertion. The examiner simply relies upon his personal knowledge.

If the examiner is contending, based on his personal knowledge, that contrast enhancement mapping disclosed in Zimmerman is **<u>identical</u>** to the contrast stretching function, this knowledge is hereby challenged. Pursuant to MPEP §707 and 37 CFR §1.104(d)(2), the examiner is respectfully requested to cite a document or affidavit supporting his personal knowledge that contrast enhancement mapping disclosed in Zimmerman is identical to the contrast stretching function. The support should include a side-by-side comparison of the steps performed during histogram equalization with the steps performed during contrast stretching. Thus far, no support or analysis has been provided. Only an unsubstantiated opinion has been provided.

<u>'103 rejection of claim 23</u>

Claim 23 recites apparatus for sharpening a digital image. Here too, the office action erroneously dismisses the weight of the preamble. The office action states a general rule for disregarding the weight of a preamble. However, MPEP 2111.02 states

> The determination of whether a preamble limits a claim is made on a case-by-case basis in light of the facts in each case; there is no litmus test defining when a preamble limits the scope of a claim.... If the claim preamble, when read in the context of the entire claim, recites limitations of the claim, or, if the claim preamble is 'necessary to give life, meaning, and vitality' to the claim, then the claim preamble should be construed as if in the balance of the claim.

The preamble of claim 23 is quite clear that the apparatus applies to sharpening of a digital image.

There is no teaching or suggestion that Zimmerman et al.'s method can be used for image sharpening. Zimmerman et al.'s method of local histogram equalization is suited for correcting uneven illumination in images. It is not suited for image sharpening because it tends to spread the local output histogram, thereby blurring edges instead of sharpening them. Therefore, claim 23 and its dependent claims 24-25 should be allowed over Burke and Zimmerman et al.

<u>'103 rejection of claim 2</u>

Claim 2 recites a method of processing pixel intensity values of a digital image. The method comprises clipping those pixel intensity values outside of a local variable range; and mapping those pixel intensity values within the local variable range. The variable range depends on minimum and maximum intensity values of a local pixel neighborhood. The mapping has a shape that depends on dynamic range of the local pixel neighborhood. The shape of the mapping function determines the type and strength of spatial filtering operation (e.g., light sharpening) that is performed.

Claim 2 recites two ranges: a dynamic range that determines the shape of a mapping function, and a variable range outside of which pixels are clipped. Zimmerman et al. do not compute any ranges.

The office action also cites Lee for pixel clipping. However, as discussed above, Lee discloses clipping on a global basis, not a local basis.

No reason exists for clipping pixel values in a histogram equalization mapping. In general, a histogram equalization mapping maps the minimum value in the histogram to itself and it maps the maximum value in the histogram to itself. Additional clipping would serve no purpose.

The office action provides a reason for modifying the teachings of Zimmerman et al. with those of Lee. The office action states "such a modification would have allowed for a method that clipped and mapped pixel intensity values on an adaptive basis rather than on the global information content of the image This would have made for a more robust system that could make objects of differing intensity value subranges simultaneously visible (Zimmerman pg. 305 left column, final paragraph)."

However, the reason is not supported by either Zimmerman et al or Lee. Zimmerman et al.'s method teaches away from mapping on a global basis. Lee, on the other hand, teaches mapping on a global basis. Moreover, neither Zimmerman et al, nor Lee explain how local mapping is more "robust" than global mapping.

For these reasons, claim 2 and its dependent claims 3 and 8-9 and 35 should be allowed over Lee and Zimmerman.

<u>Conclusion</u>

Withdrawal of the rejections is respectfully requested. The examiner is invited to contact the undersigned to discuss any issues that might remain.

Histogram-based Operations

An important class of point operations is based upon the manipulation of an image histogram or a *region* histogram. The most important examples are described below.

Contrast stretching

Frequently, an image is scanned in such a way that the resulting brightness values do not make full use of the available dynamic range. This can be easily observed in the histogram of the brightness values shown in Figure 6. By stretching the histogram over the available dynamic range we attempt to correct this situation. If the image is intended to go from brightness 0 to brightness 2^B-1 (see Section 2.1), then one generally maps the 0% value (or *minimum* as defined in Section 3.5.2) to the value 0 and the 100% value (or *maximum*) to the value 2^B-1. The appropriate transformation is given by:

$$b[m,n] = \left(2^B - 1\right) \cdot \frac{a[m,n] - \text{minimum}}{\text{maximum} - \text{minimum}}$$

This formula, however, can be somewhat sensitive to outliers and a less sensitive and more general version is given by:

$$b[m,n] = \begin{cases} 0 & a[m,n] \le p_{low}\% \\ \left(2^B - 1\right) \cdot \dfrac{a[m,n] - p_{low}\%}{p_{high}\% - p_{low}\%} & p_{low}\% < a[m,n] < p_{high}\% \\ \left(2^B - 1\right) & a[m,n] \ge p_{high}\% \end{cases}$$

In this second version one might choose the 1% and 99% values for p_{low}% and p_{high}%, respectively, instead of the 0% and 100% values represented by eq. . It is also possible to apply the contrast-stretching operation on a regional basis using the histogram from a region to determine the appropriate limits for the algorithm. Note that in eqs. and it is possible to suppress the term 2^B-1 and simply normalize the brightness range to 0 <= $b[m,n]$ <= 1. This means representing the final pixel brightnesses as reals instead of integers but modern computer speeds and RAM capacities make this quite feasible.

Equalization

When one wishes to compare two or more images on a specific basis, such as texture, it is common to first normalize their histograms to a "standard" histogram. This can be especially useful when the images have been acquired under different circumstances. The most common histogram normalization technique is *histogram equalization* where one attempts to change the histogram through the use of a function $b = f(a)$ into a histogram that is constant for all brightness values. This would correspond to a brightness distribution where all values are equally probable. Unfortunately, for an arbitrary image, one can only approximate this result.

For a "suitable" function $f(*)$ the relation between the input probability density function, the output probability density function, and the function $f(*)$ is given by:

$$p_b(b)db = p_a(a)da \quad \Rightarrow \quad df = \frac{p_a(a)da}{p_b(b)}$$

From eq. we see that "suitable" means that $f(*)$ is differentiable and that $df/da >= 0$. For histogram equalization we desire that $p_b(b)$ = constant and this means that:

$$f(a) = (2^B - 1) \bullet P(a)$$

where $P(a)$ is the probability *distribution* function defined in Section 3.5.1 and illustrated in Figure 6a. In other words, the *quantized* probability distribution function normalized from 0 to 2^B-1 *is* the look-up table required for histogram equalization. Figures 21a-c illustrate the effect of contrast stretching and histogram equalization on a standard image. The histogram equalization procedure can also be applied on a regional basis.

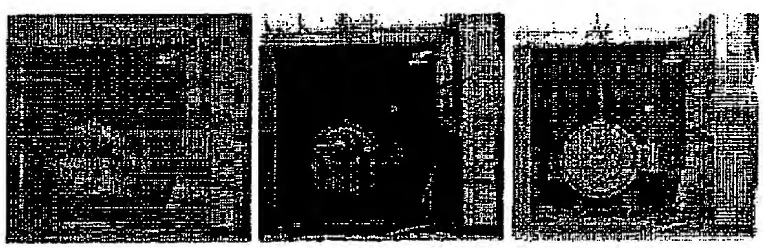
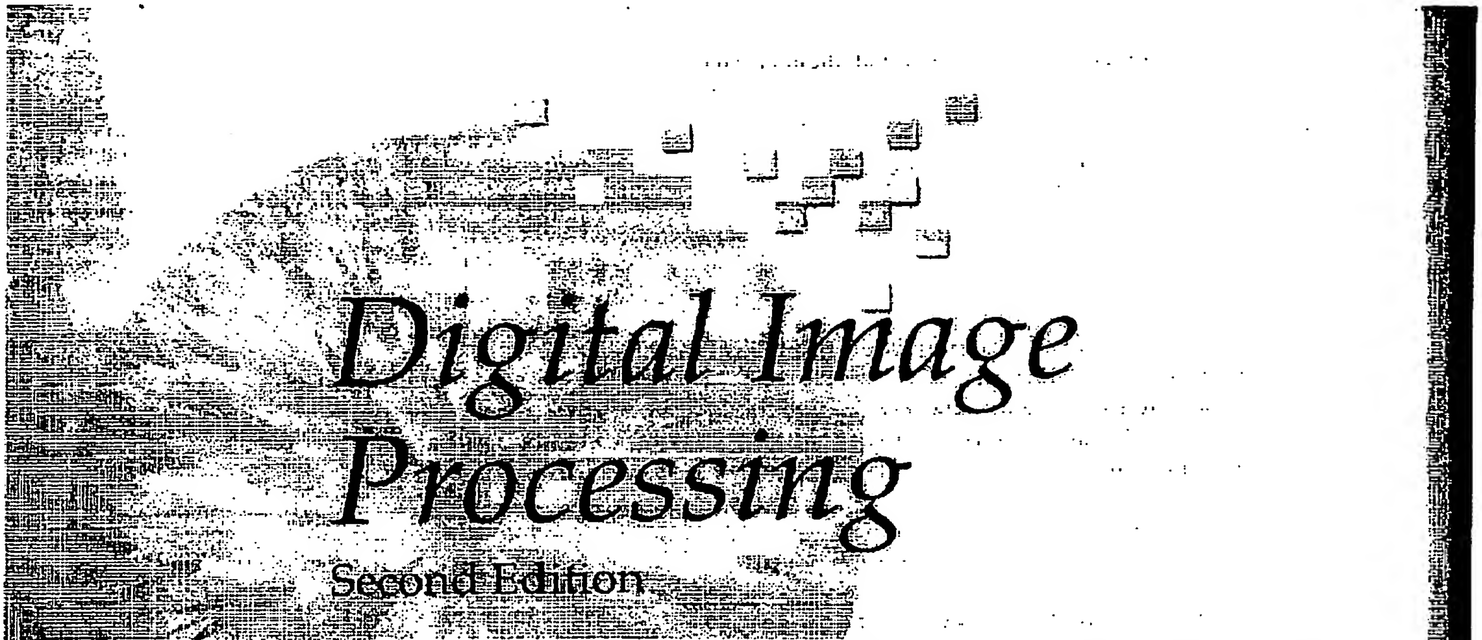


Figure 21a Figure 21b Figure 21c Original Contrast Stretched istogram Equalized

Other histogram-based operations

The histogram derived from a local region can also be used to drive local filters that are to be applied to that region. Examples include *minimum* filtering, *median* filtering, and *maximum* filtering . The concepts minimum, median, and maximum were introduced in Figure 6. The filters based on these concepts will be presented formally in Sections 9.4.2 and 9.6.10.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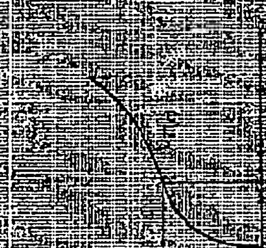

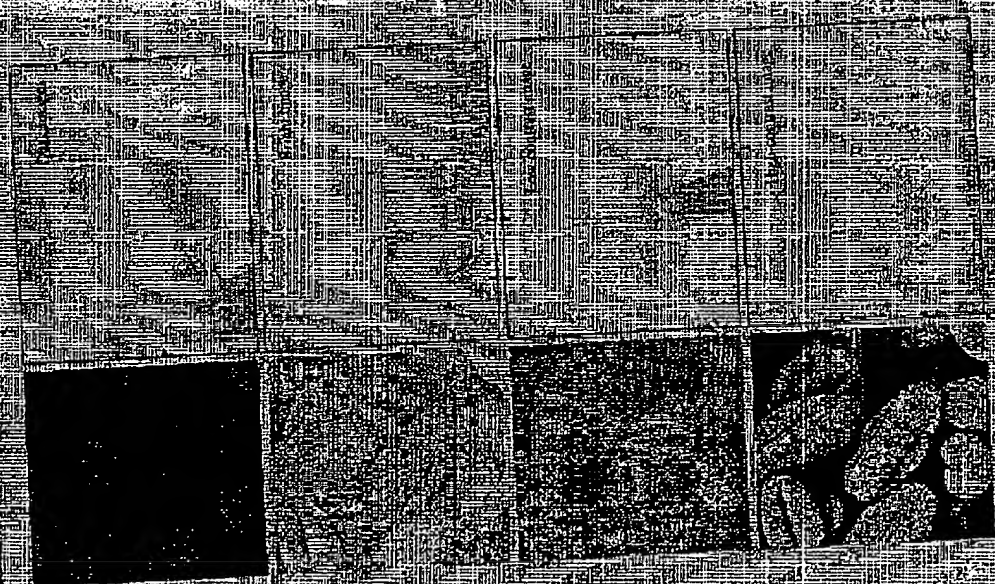

The gray levels in an image may be viewed as random variables in the interval [0, 1]. One of the most fundamental descriptors of a random variable is its probability density function (PDF). Let $p_r(r)$ and $p_s(s)$ denote the probability density functions of random variables r and s, respectively, where the subscripts on p are used to denote that p_r and p_s are different functions. A basic result from an elementary probability theory is that, if $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies condition (a), then the probability density function $p_s(s)$ of the transformed variable s can be obtained using a rather simple formula:

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| \tag{3.3-3}$$

Thus, the probability density function of the transformed variable, s, is determined by the gray-level PDF of the input image and by the chosen transformation function.

A transformation function of particular importance in image processing has the form

$$s = T(r) = \int_0^r p_r(w)\,dw \tag{3.3-4}$$

where w is a dummy variable of integration. The right side of Eq. (3.3-4) is recognized as the cumulative distribution function (CDF) of random variable r. Since probability density functions are always positive, and recalling that the integral of a function is the area under the function, it follows that this transformation function is single valued and monotonically increasing, and, therefore, satisfies condition (a). Similarly, the integral of a probability density function for variables in the range [0, 1] also is in the range [0, 1], so condition (b) is satisfied as well.

Given transformation function $T(r)$, we find $p_s(s)$ by applying Eq. (3.3-3). We know from basic calculus (Leibniz's rule) that the derivative of a definite integral with respect to its upper limit is simply the integrand evaluated at that limit. In other words,

$$\begin{aligned}\frac{ds}{dr} &= \frac{dT(r)}{dr}\\ &= \frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right]\\ &= p_r(r).\end{aligned} \tag{3.3-5}$$

Substituting this result for dr/ds into Eq. (3.3-3), and keeping in mind that all probability values are positive, yields

$$\begin{aligned}p_s(s) &= p_r(r)\left|\frac{dr}{ds}\right|\\ &= p_r(r)\left|\frac{1}{p_r(r)}\right|\\ &= 1 \qquad 0 \le s \le 1.\end{aligned} \tag{3.3-6}$$

Because $p_s(s)$ is a probability density function, it follows that it must be zero outside the interval [0, 1] in this case because its integral over all values of s must equal 1. We recognize the form of $p_s(s)$ given in Eq. (3.3-6) as a *uniform* probability density function. Simply stated, we have demonstrated that performing the transformation function given in Eq. (3.3-4) yields a random variable s characterized by a uniform probability density function. It is important to note from Eq. (3.3-4) that $T(r)$ depends on $p_r(r)$, but, as indicated by Eq. (3.3-6), the resulting $p_s(s)$ *always* is uniform, *independent* of the form of $p_r(r)$.

For discrete values we deal with probabilities and summations instead of probability density functions and integrals. The probability of occurrence of gray level r_k in an image is approximated by

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0, 1, 2, \ldots, L-1 \tag{3.3-7}$$

where, as noted at the beginning of this section, n is the total number of pixels in the image, n_k is the number of pixels that have gray level r_k, and L is the total number of possible gray levels in the image. The discrete version of the transformation function given in Eq. (3.3-4) is

$$\begin{aligned}s_k = T(r_k) &= \sum_{j=0}^{k} p_r(r_j)\\ &= \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, 1, 2, \ldots, L-1.\end{aligned} \tag{3.3-8}$$

Thus, a processed (output) image is obtained by mapping each pixel with level r_k in the input image into a corresponding pixel with level s_k in the output image via Eq. (3.3-8). As indicated earlier, a plot of $p_r(r_k)$ versus r_k is called a *histogram*. The transformation (mapping) given in Eq. (3.3-8) is called *histogram equalization* or *histogram linearization*. It is not difficult to show (Problem 3.9) that the transformation in Eq. (3.3-8) satisfies conditions (a) and (b) stated previously in this section.

Unlike its continuous counterpart, it cannot be proved in general that this discrete transformation will produce the discrete equivalent of a uniform probability density function, which would be a uniform histogram. However, as will be seen shortly, use of Eq. (3.3-8) does have the general tendency of spreading the histogram of the input image so that the levels of the histogram-equalized image will span a fuller range of the gray scale.

We discussed earlier in this section the many advantages of having gray-level values that cover the entire gray scale. In addition to producing gray levels that have this tendency, the method just derived has the additional advantage that it is fully "automatic." In other words, given an image, the process of histogram equalization consists simply of implementing Eq. (3.3-8), which is based on information that can be extracted directly from the given image, without the need for further parameter specifications. We note also the simplicity of the computations that would be required to implement the technique.

The inverse transformation from s back to r is denoted by

$$r_k = T^{-1}(s_k) \qquad k = 0, 1, 2, \ldots, L-1 \tag{3.3-9}$$

It can be shown (Problem 3.9) that the inverse transformation in Eq. (3.3-9) satisfies conditions (a) and (b) stated previously in this section only if none of the levels, r_k, $k = 0, 1, 2, \ldots, L-1$, are missing from the input image. Although the inverse transformation is not used in histogram equalization, it plays a central role in the histogram-matching scheme developed in the next section. We also discuss in that section details of how to implement histogram processing techniques.

EXAMPLE 3.3: Histogram equalization.

■ Figure 3.17(a) shows the four images from Fig. 3.15, and Fig. 3.17(b) shows the result of performing histogram equalization on each of these images. The first three results (top to bottom) show significant improvement. As expected, histogram equalization did not produce a significant visual difference in the fourth image because the histogram of this image already spans the full spectrum of the gray scale. The transformation functions used to generate the images in Fig. 3.17(b) are shown in Fig. 3.18. These functions were generated from the histograms of the original images [see Fig. 3.15(b)] using Eq. (3.3-8). Note that transformation (4) has a basic linear shape, again indicating that the gray levels in the fourth input image are nearly uniformly distributed. As was just noted, we would expect histogram equalization in this case to have negligible effect on the appearance of the image.

The histograms of the equalized images are shown in Fig. 3.17(c). It is of interest to note that, while all these histograms are different, the histogram-equalized images themselves are visually very similar. This is not unexpected because the difference between the images in the left column is simply one of contrast, not of content. In other words, since the images have the same content, the increase in contrast resulting from histogram equalization was enough to render any gray-level differences in the resulting images visually indistinguishable. Given the significant contrast differences of the images in the left column, this example illustrates the power of histogram equalization as an adaptive enhancement tool. ■

3.3.2 Histogram Matching (Specification)

As indicated in the preceding discussion, histogram equalization automatically determines a transformation function that seeks to produce an output image that has a uniform histogram. When automatic enhancement is desired, this is a good approach because the results from this technique are predictable and the method is simple to implement. We show in this section that there are applications in which attempting to base enhancement on a uniform histogram is not the best approach. In particular, it is useful sometimes to be able to specify the shape of the histogram that we wish the processed image to have. The method used to generate a processed image that has a specified histogram is called *histogram matching* or *histogram specification*.

Development of the method

Let us return for a moment to continuous gray levels r and z (considered continuous random variables), and let $p_r(r)$ and $p_z(z)$ denote their corresponding continuous probability density functions. In this notation, r and z denote

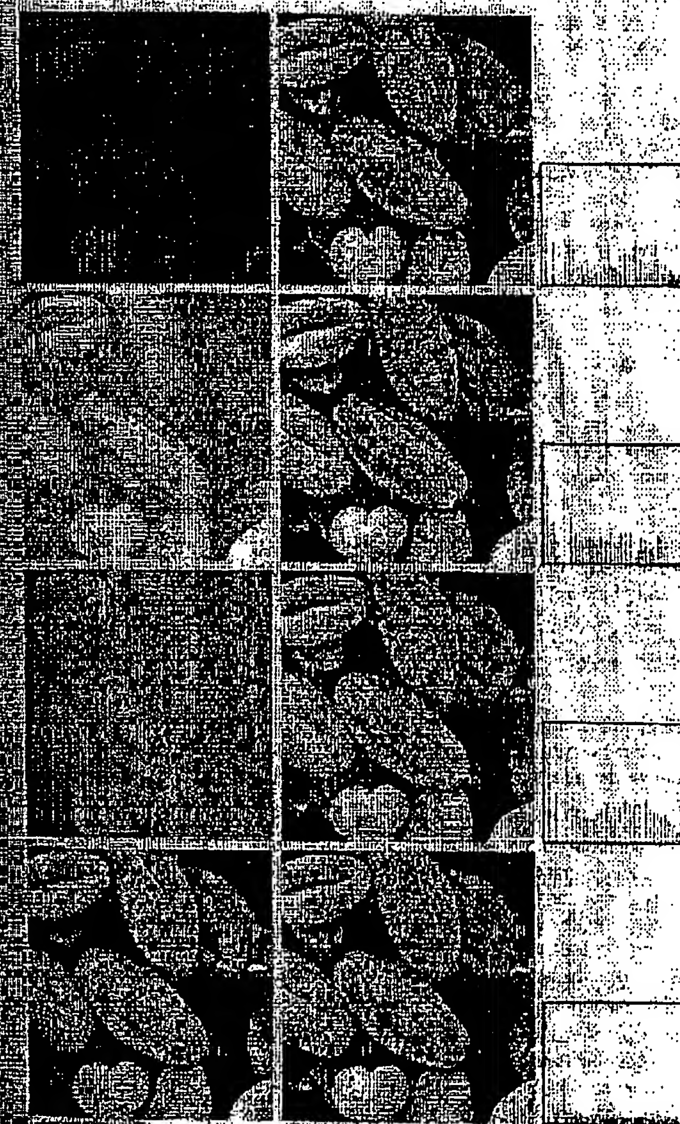

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.